ECON 301, Professor Hogendorn, Spring 2015

Final Exam Take-Home Mathematica Project

Total of 4 5-point questions, +1 or +2 extra credit possible.

Please turn in a Mathematica print-out at the sit-down final.

1. *ProductiveFirms.* There are two types of firms. Type 1 firms are more productive and have production function $f(L) = \frac{1}{2}L^{\frac{1}{2}}$. Type 2 firms are less productive and have production function $f(L) = \frac{1}{4}L^{\frac{1}{2}}$. These two firm types supply a perfectly competitive market with demand curve $y^d = 7 - p$.

   The number of type-1 firms is equal to the last digit of your WesID. The number of type-2 firms is equal to the second-to-last digit of your WesID. If either digit is 0, move to the left so that there is a positive number of both types of firms.

   These firms hire labor in a market with labor supply $L^S = 2w$, where w is the wage. The labor market is also perfectly competitive.

   Answer the following questions using Mathematica. There is a hint notebook following this question to help you think about how to do this. But note that the hint notebook is NOT all the way filled in, and it's for someone whose WesID ends in 74. You will have to fill in the gaps and change the firm numbers.

   (a) What is the equilibrium price and quantity in the goods market (as a function of the wage)?

   (b) What is the equilibrium wage in the labor market?

   (c) Graph the labor market, showing the total labor demand from all firms, the total labor demand from type 1 firms, and the total labor demand from type 2 firms.

(d) Would it raise the wage more if there were 1 more firm of type 1 or type 2? You don't have to show the output, just rerun your notebook using the new numbers and report the resulting wage. More important, give an intuitive explanation.

(e) *Bonus +2 points.* If the workers could form a union, and thus sell their labor as a monopoly, what would be the monopoly optimal wage set by the union?

(f) *Additional interesting question, but no points, just for fun.* If the monopolist union could wage-discriminate between the two types of firms, would it set a higher wage for the type 1 firm or the type 2 firm?

Two Types of Firms, Type 1 is More Productive

These are the only primitives of this whole problem:

Let labor supply be

```
Ls = 2 * w
```

2 w

Let type 1 firms have production function
$f(L) = \frac{1}{2} L^{1/2}$,
this will mean that conditional labor demand is $L1(y) = 4 y^2$.

```
L1[y_] := 4 * y^2
```

Let type 2 firms have production function $f(L) = \frac{1}{4} L^{1/2}$,
this will mean that conditional labor demand is $L1(y) = 16 y^2$.

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L2[y_] := 16 * y^2
```

Let demand in this market be

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yd = 7 - p
```

7 − p

Find Supply Curve

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TVC1 = w * L1[y]
```

$4 w y^2$

```
MC1 = D[TVC1, y]
```

8 w y

Find the type 1 supply curve by setting MC(y)=p and solving for y:

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ys1 = y /. Solve[p == MC1, y][[1]]
```

$\dfrac{p}{8\,w}$

Find Equilibrium in Goods Market

Set demand equal to supply to get market equilibrium price:

```
pstar = p /. Solve[yd == 4 * ys1 + 7 * ys2, p][[1]]
```

$\dfrac{224\,w}{23 + 32\,w}$

And substitute back into demand to find the equilibrium quantity:

Find the outputs of both firm types

Find Equilibrium in Labor Market

Find the labor demand of both firm types

Find total labor demand and equilibrium wage

Graph the Labor Market, Including Total Demand from Both Firm Types

```
Plot[{inverselabordemand, inverselabordemand1, inverselabordemand2,
   inverselaborsupply, wstar}, {L, 0, 10}, PlotLegends → "Expressions"]
```