

ECON 301, Prof. Hogendorn

Problem Set 1

1. *MRS*. Let an individual have the utility function

$$u(x, y) = x^{1/3} y^{2/3}$$

Use the standard space with  $x$  on the horizontal axis and  $y$  on the vertical axis for the following:

- (a) Compute the marginal rate of substitution when  $x = y = 8$ . Draw on a graph.
  - (b) Compute the marginal rate of substitution when  $x = 512$  and  $y = 1$ . Why is it different from part (a)? What does this explain about the person's preferences?
  - (c) Use the total differential to estimate the change in utility from starting at  $x = y = 8$  and moving to  $x = 512, y = 1$ . (Use the point  $(8,8)$  to evaluate the partial derivatives.)
  - (d) The estimate from part (c) is much larger than the actual change in utility. Why? Since it's wrong, does it mean that differentials are not very useful in real life?
2. *Albums*. In 2012, sales of digital music albums were 118 (million) and sales of vinyl albums were 4.5. In 2013, sales of digital albums were the same at 118 and sales of vinyl albums were 6.

Let's assume it's the "representative consumer" who buys these albums, so they're all in one indifference curve / budget line diagram. Assume that in both years the consumer makes a utility-maximizing decision.

Consider each of the following changes in isolation. Are both of them consistent with the utility maximizing model and the consumption pattern described above? Illustrate on two separate diagrams with digital on the vertical axis and vinyl on the horizontal.

- (a) Price of vinyl declined and price of digital went up.
- (b) Price of vinyl declined and price of digital stayed the same.

3. *DanBrown*. Amazon sells the bestselling novel *Inferno* in different formats. Rounding the prices to the nearest dollar, the e-book edition on the Amazon Kindle costs \$15 and the hardcover print edition costs \$18.

There are two types of consumers, affectionately called Inkies and Pixlees. Each type of consumer has \$20 of income that they may allocate between the two types of books. (Don't worry about getting answers with fractional books.)

- (a) Write down the budget constraint for buying  $k$  units of the Kindle edition (horizontal axis) versus  $h$  units of the hardcover edition (vertical axis). Label the vertical and horizontal intercepts.
- (b) Find the slope of the budget constraint by using the total differential.
- (c) Let Pixlees have utility function:

$$u(k, h) = k^{0.9}h^{0.1}$$

What is a Pixlee's marginal rate of substitution? What is their utility maximizing consumption of  $k$  and  $h$ ?

- (d) Inkies have a quasilinear utility:

$$u(k, h) = (k + 1)^{0.5} + h$$

What do their indifference curves look like? Hint: what is the MRS when  $k=0$ ?

- (e) How many books of each type do Inkies buy when they maximize utility?

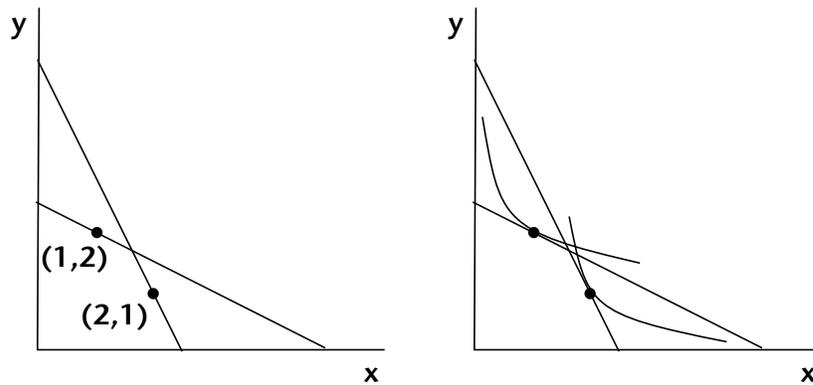
### Review Problems Only, Not to Turn In:

4. *Consistent.* When prices are  $p_x = 1, p_y = 2$ , a consumer demands  $x = 1, y = 2$ , and when prices are  $p_x = 2, p_y = 1$ , the consumer demands  $x = 2, y = 1$ . Is this behavior consistent with the model of utility maximizing behavior? Draw an indifference curve / budget line diagram to illustrate your answer.
5. *Buying X and Y.* Suppose you have an income of \$40 to spend on two commodities. Commodity X costs \$10 per unit and commodity Y costs \$5 per unit.
- Write down your budget constraint. If you spent all your income on X, how much could you buy? If you spent all your income on Y, how much could you buy? Graph your budget constraint. What is its slope?
  - Suppose the price of X falls to \$5 while everything else stays the same. Write down your new budget constraint. What is its slope? Graph your new budget constraint on the same graph as (a).
  - Suppose your income falls to \$30, but the prices of X and Y remain at \$5. Write down your new budget constraint. What is its slope? Graph your new budget constraint on the same graph as (a) and (b).
  - On your graph, shade in the area representing commodity bundles that you can afford with the budget in (c) but could not afford to buy with (a). Shade in the area representing commodity bundles that you could afford with the budget in (a) but cannot afford with the budget in (c).

6. *Jazz*. Suppose that there are two types of jazz music, smooth jazz and traditional jazz. The only jazz radio station in a city plays 10 minutes of smooth jazz for every 10 minutes of traditional jazz. Graph this point (10,10), and then analyze the preferences of the listeners. Assume all listeners have convex preferences, and there are two *equal-sized* groups of listeners, Group 1 and Group 2.
- (a) At the (10,10) point, Group 1 listeners have an MRS of 1 minute of traditional jazz per 4 minutes of smooth jazz. Graph the group 1 indifference curve through the (10,10) point.
  - (b) At the (10,10) point, Group 2 listeners have an MRS of 8 minutes of traditional jazz per 1 minutes of smooth jazz. Graph the group 2 indifference curve through the (10,10) point.
  - (c) Suppose the radio station changes its format somewhat and plays 11 minutes of smooth jazz for each 9 minutes of traditional jazz. Which type of listener has moved further in space on the graph from its old indifference curve?
  - (d) Can we say which type of listener is better off and which worse off? Can we say whether the gains to one group more than offset the losses to the other? (Be very careful on this last question.)

## Answers to Review Problems:

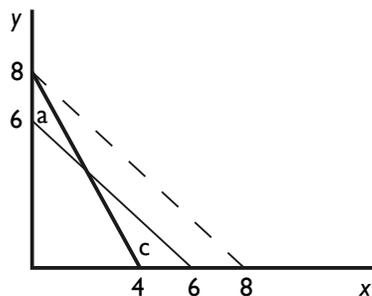
4. *Consistent\_a*. First, just draw the budget lines and label the points.



Now if it is really true that both of these points represent utility maximums, it must be that both are tangent to indifference curves. But this means the indifference curves must cross, since they are tangent to lines of different slopes. If indifference curves cross, then preferences are not monotonic – from some points, more of both goods would lower utility.

5. *BuyingXandY\_a*

- (a)  $10X + 5Y = 40$ . If you spent all your income on X, you could buy 4. If you spent all your income on Y, you could buy 8. The slope is  $-8/4 = -2$ . This is represented by the dark budget line in the following graph.



- (b)  $5X + 5Y = 40$ .  $Y = 8 - X$ . Slope  $-1$ . Dashed line in graph.
- (c)  $5X + 5Y = 30$ .  $Y = 6 - X$ . Slope  $-1$ . Narrow solid line in graph.
- (d) Can afford area c with budget (c) but not (a). Can afford area a with budget (a) but not (c).

6. *Jazz\_a*.

- (a) A nice convex indifference curve.
- (b) The curves cross. Group 2's is steeper.
- (c) If a group had an MRS of  $-1$ , then that group would be indifferent to this movement. Both groups have MRSs that differ from  $-1$ : group 1's slope is 4 times less and group 2's slope is 8 times more. Thus Group 2's slope differs more and it will therefore move further in space from its old indifference curve (at least using the calculus approximation that is inherent in MRS).
- (d) Group 1 is below its old indifference curve and is therefore worse off. Group 2 is above its old indifference curve and therefore better off. We can't compare utility measures between people, so we can't say whether the gains more than offset the losses.