

ECON 301, Professor Hogendorn

Problem Set 2

1. *Electricity.* Suppose you want to study electricity demand. You might conclude that most families' tradeoff between e (electricity) and x (the numeraire) is independent of m . In this case, a quasilinear utility function is appropriate, along with a standard budget line.

$$\begin{aligned}u(e, x) &= w(e) + x \\ p_e e + x &= m\end{aligned}$$

- (a) Set up the Lagrangian for this problem and take the first order conditions. (Note that $w(e)$ is some function you do not know explicitly).
- (b) What is the demand function for e and the value for λ at the optimum?
- (c) Explain what the value of λ means and why the particular value you found makes sense.
- (d) Is there anything else you should check to make sure that the demand function for e is reasonable? Hint: what if income were very low?
2. *Budget.* There are two goods, both of which are provided by the government. One is defense and homeland security spending (d), and the other is all other government discretionary functions (g) (environmental protection, transportation, health and human services, etc., but not including social security, Medicare, or welfare). Both are measured in billions of dollars, so the price of each good is 1 billion.

Suppose the proposed budget is $d = 472$ and $g = 398$. If we take the sum of these two as given, we can call the government's "income" $m = 870$.

Consider a Senator with the utility function

$$u(G, D) = 2G + DG$$

- (a) If this Senator could decide the government budget allocation herself, what would she pick? Use the Lagrangian to show that $G^* = 436$ and $D^* = 434$.
- (b) Using the total differential of the utility function, estimate the change in utility of this Senator if she were able to change the budget to her preferred point.
- (c) Draw an accurate graph of the two points, showing the budget line and indifference curves.
- (d) Now suppose that the Senator would have to "pay" for the privilege of choosing the budget allocation by a reduction in m . (For example, the Senator might have to offer tax cuts to get her budget passed, and these tax cuts would reduce the m available.) Use the Lagrange multiplier to determine the largest reduction in m the Senator would tolerate.
- (e) This Senator is fairly middle-of-the-road in political views. Suppose you want to model a more liberal Senator, one who would prefer much more G than D . Should you change the utility function to $u(G, D) = 0.0002G + DG$ or $u(G, D) = 200G + DG$. Explain your answer with reference to the concepts of marginal utility and marginal rate of substitution.

3. *Medical*². Prices of medical services have been rising much faster than other goods and services in the economy. Let μ be medical services and x be all other goods. Suppose that a consumer has a

demand curve for medical services of

$$\mu(p_\mu, p_x, m) = \frac{m}{4.5p_\mu}$$

- (a) In 2007, the prices were $p_x = 1$, $p_\mu = 1$, and $m = 54.5$. By 2011 prices had risen to $p'_x = 1.08$, $p'_\mu = 1.12$ and income had fallen to $m' = 50.1$. Draw an indifference curve diagram, (with x on the x-axis) showing the two budget lines and the two optimal points. Remember that all income not spent on μ is spent on x .
- (b) Calculate the Laspeyres price index for the price change from 2007 to 2011.
- (c) Calculate the Paasche price index for the price change from 2007 to 2011.
- (d) If the consumer had been given a raise based on the Laspeyres price index, how much x and μ would she have consumed in 2011. Would her utility have been higher or lower than in 2007?

Review Problems, not to turn in:

4. *Urp*. The residents of Urp consume only pork chops (X) and Coca-Cola (Y). The utility function of the typical resident is given by

$$U(X, Y) = \sqrt{XY}$$

In 2006, the price of pork chops in Urp was \$1 each; Cokes were also \$1 each. The typical resident consumed 40 pork chops and 40 Cokes (saving is impossible in Urp). In 2007, swine fever hit Urp, and pork chop prices rose to \$4; the Coke price remained unchanged. At these new prices, the typical Urp resident consumed 20 pork chops and 80 Cokes.

- (a) What was the change in utility from 2006 to 2007? (Just plug into the utility function, don't use differentials.)
- (b) What was the Laspeyres price index for 2007?
- (c) What was the Paasche price index for 2007?
- (d) What do you conclude about the ability of price indices to measure changes in welfare? (Hint: calculate how much income the typical Urp resident had in 2006 and 2007.)

5. *Martini*. The Martini is a famous cocktail that is properly made with gin and vermouth. (Vodka martinis are a horrible travesty from the 1960s and 70s.) Let your utility function be

$$u(G, V) = G^{0.9} V^{0.1}$$

where G is ounces of gin and V is ounces of vermouth. Let the price of gin be \$1 per ounce and the price of vermouth be \$0.40 per ounce.

- (a) Using only the information above, describe what proportions you use to make a martini.
- (b) Now suppose that you have \$4 available to spend on martinis. Use the Lagrangian to solve the utility maximization problem, and then show how many total ounces are in the drink you make.
- (c) If you had one more dollar, how much additional utility would you receive?

6. *Lambda*. Suppose that a consumer has utility function $u(x, y) = x^{1/3} y^{2/3}$. The income is m , and the prices of the goods are both 1. Use the Lagrangian to solve for the value of λ . Then find $\partial u(x^*, y^*) / \partial m$ where x^* and y^* are the optimal solutions that come out of the Lagrangian. What is the relationship between λ and $\partial u(x^*, y^*) / \partial m$?

Answers to Review Problems:

4. *Urp_a*.

(a) $U_{2006} = \sqrt{40 \cdot 40} = 40$, $U_{2007} = \sqrt{20 \cdot 80} = 40$

(b) $\frac{4 \cdot 40 + 1 \cdot 40}{1 \cdot 40 + 1 \cdot 40} = 2.5$

(c) $\frac{4 \cdot 20 + 1 \cdot 80}{1 \cdot 20 + 1 \cdot 80} = 1.6$

(d) We know that in actual fact, utility was unchanged and the new income in 2007 must have been $4 \cdot 20 + 1 \cdot 80 = 160$ which was twice the income of $1 \cdot 40 + 1 \cdot 40 = 80$ in 2000. Thus, Laspeyres overstated the amount of income needed to keep utility constant, and Paasche understated it.

5. *Martini_a*

(a)

$$MRS = -\frac{\frac{\partial u}{\partial G}}{\frac{\partial u}{\partial V}} = -\frac{.9G^{-.1}V^{.1}}{.1G^{.9}V^{-.9}} = -9\frac{V}{G}$$

Now set MRS equal to the slope of the budget line:

$$-9\frac{V}{G} = -\frac{1}{.4} = -\frac{10}{4} \Rightarrow G = \frac{36}{10}V \Rightarrow G = 3.6V$$

So use 3.6 parts gin to one part vermouth.

(b)

$$\begin{aligned}\max_{G,V,\lambda} \mathcal{L} &= G^{.9}V^{.1} - \lambda(G + .4V - 4) \\ \frac{\partial \mathcal{L}}{\partial G} &= .9G^{-.1}V^{.1} - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial V} &= .1G^{.9}V^{-.9} - .4\lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= G + .4V - 4 = 0\end{aligned}$$

Solving simultaneously we get:

$$\lambda = \lambda \Rightarrow G = 3.6V \text{ (We already found this in part a)}$$

$$3.6V + .4V - 4 = 0$$

$$4V = 4 \Rightarrow V = 1 \quad G = 3.6$$

So you make a martini using 1 ounce of vermouth and 3.6 ounces of gin, so the total amount of liquid in the drink is 4.6 ounces.

$$(c) \lambda = .9G^{-.1}V^{.1} = .9 \cdot 3.6^{-.1} \cdot 1 = .79$$

6. *Lambda_a.*

$$\max_{x,y,\lambda} \mathcal{L} = x^{1/3}y^{2/3} - \lambda(x + y - m)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{3}x^{-2/3}y^{2/3} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{2}{3}x^{1/3}y^{-1/3} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = x + y - m = 0$$

Solving simultaneously we get:

$$\lambda = \lambda \Rightarrow y = 2x$$

$$x + 2x - m = 0$$

$$x = \frac{m}{3}, y = \frac{2m}{3}, \lambda = \frac{1}{3}x^{-2/3}y^{2/3} = 0.529$$

Now subbing the optimal x, y into the utility function gives:

$$U(x^*, y^*) = \left(\frac{m}{3}\right)^{1/3} \left(\frac{2m}{3}\right)^{2/3} = 0.529m$$

From this it is clear that $\partial u(x^*, y^*)/\partial m = 0.529 = \lambda$.