ECON 301, Professor Hogendorn

Problem Set 4

1. *Laffer.* The "Laffer Curve" first became famous in the Reagan administration. It shows that when taxes are high enough, raising taxes can actually *reduce* tax revenue. While the theory is sound, its application to the U.S. income tax did not raise revenue under Reagan or either Bush.

   Suppose there is a tax t so that consumer pay price p and sellers receive price $p - t$. Demand is $x(p)$, and supply is $s(p - t)$.

   (a) Graph the tax and show the deadweight loss. Explain in words the deadweight loss.

   (b) What is the change in equilibrium price when the tax changes? I.e., what is $\frac{dp}{dt}$? Express this in terms of elasticities as much as possible.

   (c) Now since tax revenue is $TR = tx(p)$, find $\frac{dTR}{dt}$. Put it in elasticity form as much as possible.

   (d) Is it possible that raising the tax could reduce the revenue the government receives? Prove or disprove by signing the derivative from (c).

   (e) Explain in words the logic of part (d). Make sure you use the word "elasticity" in your answer.

2. *Snowboards.* There has been a recent collapse of interest in snowboarding. This is because of advances in ski technology that make skiing more fun for many people.

   (a) Draw a graph of the supply and demand of snowboards. Show what happens as a result of the decline in peoples' interest in

snowboards. Label the old and new equilibrium quantities and prices.

(b) Is there deadweight loss associated with the change in part (a)? Explain why or why not, and if there is, show it in the diagram.

(c) Now let's be more specific. Snowboard market demand is $x(p) = a - p$ and snowboard market supply is $s(p) = 3p$. Find an expression for $\frac{dp}{da}$. Prove whether $\frac{dp}{da}$ is positive or negative.

(d) And now a slightly harder version. Let's say we don't actually know the specific functional forms of demand. All we know is that there is some demand function $x(p, a)$ that is decreasing in p and increasing in a. And we know there is some supply function $s(p)$. And of course we know that supply equals demand. Using the total differential, find an expression for $\frac{dp}{da}$. Try to write the expression in elasticity form (but it won't be as neat as some of our other examples, you'll still have some extraneous stuff). Prove that the expression is positive.

(e) Bonus +1 point, don't play around with this unless you have extra time. In (d), you could find an expression for the elasticity of p with respect to a. If you do that, you can make the entire expression a relationship between various elasticities.

3. *EducatedMothers.* A strong finding by development economists around the world is that when women are better educated, not only does their own standard of living rise, but so does their children's. The effect is largely due to the education of the mother herself, but also the average educational level of women in the community makes a difference. Suppose for example that a typical woman's utility function is $u_w(x, e) = x^{3/4}e^{1/4}$ where e is her own educational level and that a typical child's utility function is $u_c(e, E) = 12e^{1/16}E^{3/16}$ where e is the child's mother's education

level and E is the average educational level of other women. Let there be one thousand women in the community. Note the child just takes x, e, and E as given.

(a) Is there a positive externality in consumption of e? How does it operate? Do you expect that this externality will be internalized in any way? Intuitively (no math), what is the difference between the free-market e and the socially optimal levels?

(b) Suppose the price of x is $p_x = 1$ and the price of e is p_e. What is the woman's MRS in (x, e) space? If the woman's income is 2,000, what is her private demand curve for education?

(c) Suppose a social planner cares equally about women and children and that each woman has exactly three children. What is a social planner's MRS in (x, e) space? If the the a typical woman's income is 2,000, what is the social demand curve for education? Graph the two curves. What Pigouvian subsidy would correct the externality?

Review Problems, not to turn in:

4. *Levin.* This question is inspired by Senator Carl Levin's report on gas prices (April 2002).

(a) There is a rule of thumb in the oil industry that each 10 cent increase in the price of gas adds \$10 billion to oil industry revenues. This implies that

$$0.10 \frac{dTR}{dp} = 10,000,000,000$$

Show that you can obtain an elasticity estimate of $\epsilon = -0.23$ from this formula if you also know that the total quantity of gas consumed per year is 130 billion gallons.

(b) The average American spends $1,060 per year on gas and consumes 700 gallons. Let us suppose that the average American has an income of $m = 50,000$. Suppose you want to calibrate a demand curve of the following form :

$$y(p) = Amp^\epsilon$$

Show that $A = 0.0154$.

(c) Perhaps we have chosen a bad demand function. Consider the following two demand functions:

$$\begin{aligned} y(p) &= A\sqrt{m}p^\epsilon \\ y(p) &= Am^2p^\epsilon \end{aligned}$$

Draw the Engel curves that correspond to these functions. Which one is more reasonable for gas?

5. *Thornton.* Suppose the mayor of Middletown proposes a new tax on restaurant meals to finance Main Street improvements. Restaurant meals are elastically supplied at $s(p_s) = -5600 + 400p_s$. The tax will be a per unit tax, so the price restauranteurs receive is p_s and the price diners must pay is $p = p_s + t$. Demand for restaurant meals is $x(p) = 500 - 3p$.

(a) Show the equilibrium price and quantity without the tax are $15.14 and 456 respectively. Find the demand and supply elasticities at this equilibrium, and explain (in words) who will pay the tax, producers or consumers?

(b) Show that the change in p_s when there is a change in the tax is 0.00744. Use the total derivative of the equilibrium condition.

(c) Find a formula for $p_s(t)$, the equilibrium producer price given a tax of t. Then find formulas for $S(p_s(t))$, government revenue, and for deadweight loss as functions of t. The changes

4

in government revenue and deadweight loss are respectively:

$$\frac{dR}{dt} = 456 - 5.96t \qquad \frac{dDWL}{dt} = 2.98t$$

(d) Is it possible for the mayor to get in a situation where he or she cannot raise enough tax revenue to fund the improvements without causing more deadweight loss than the gains to Middletown from having the improvements? Explain with reference to the above formulas and to the elasticities of supply and demand.

Answer to Review Problems:

4. *Levin_a.*

(a) We find this by expanding the derivative of total revenue and manipulating it to get the elasticity formula:

$$
\begin{aligned}
0.10\frac{dTR}{dp} &= 10,000,000,000 \\
0.10\left(y + p\frac{dy}{dp}\right) &= 10,000,000,000 \\
0.10y\left(1 + \frac{p}{y}\frac{dy}{dp}\right) &= 10,000,000,000 \\
(1 + \epsilon) &= \frac{100,000,000,000}{y} \\
\epsilon &= \frac{100,000,000,000}{130,000,000,000} - 1 \\
\epsilon &= 0.77 - 1 \\
\epsilon &= -0.23
\end{aligned}
$$

(b) First, the average price of gas must be $\frac{1060}{700} = 1.51$. Given that, we need to fit the demand curve:

$$
\begin{aligned}
700 &= A \cdot 50,000 \cdot 1.51^{-0.23} \\
0.014 &= A \cdot 0.91 \\
0.0154 &= A
\end{aligned}
$$

5

(c) The Engel curves depend on the m term only, and look like:

$$y=Am^{1/2}p^{-0.23} \qquad y=Am^2p^{-0.23}$$
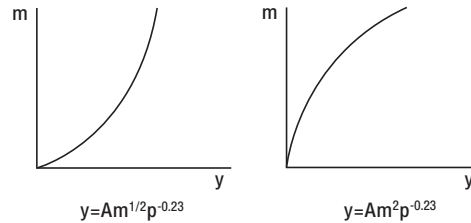
(remember, Engel curves are graphed backwards, just like demand curves, so y is the dependent variable even though it's on the horiztonal axis.) The first function is probably more reasonable, because we would expect gas to be a necessity, rising with income but not at as great a rate. Of course, up to a point, SUVs, ATVs, and boats might make gas a luxury.

5. *Thornton_a.*

(a) Setting demand equal to supply gives:

$$500 - 3p_s = -5600 + 400p_s \Rightarrow p_s = 15.14, \quad s(15.14) = 456$$

The elasticities are $\epsilon = -0.1$ and $\epsilon_s = 13.28$.

(b) The equilibrium condition is

$$500 - 3(p_s + t) = -5600 + 400p_s$$

The total derivative is then

$$-3\left(\frac{dp_s}{dt} + 1\right) = 400\frac{dp_s}{dt}$$

We can solve this for

$$\frac{dp_s}{dt} = -0.00744$$

(c)

$$500 - 3(p_s + t) = -5600 + 400p_s$$

$$6100 - 3t = 403p_s$$

$$p_s = 15.14 - \frac{3}{403}t$$

$$S(p_s) = -5600 + 400(15.14 - \frac{3}{403}t) = 456 - 2.98t$$

$$\frac{dR}{dt} = \frac{dtS(p_s)}{dt}$$

$$= \frac{d456t - 2.98t^2}{dt} = 456 - 5.96t$$

$$\frac{dDWL}{dt} = \frac{d\frac{1}{2}(456 - s(p_s))t}{dt}$$

$$= \frac{d\frac{1}{2}(456 - 456 + 2.98t)t}{dt}$$

$$= \frac{d1.49t^2}{dt} = 2.98t$$

(d) We know supply is very elastic and demand very inelastic. That means that adding a tax will basically increase the price to consumers a lot. As the tax increases, the marginal government revenue added goes down while the deadweight loss rises. Eventually, you reach a tax such that

$$\frac{dR}{dt} = 456 - 5.96t = 2.98t = \frac{dDWL}{dt} \Rightarrow t = 51$$

So, once the tax reaches 51, each marginal increase in tax causes more DWL than it does tax revenue.