ECON 301, Professor Hogendorn

Problem Set 5

1. *Sigma.* Consider the production function

$$f(L,K) = (L^{0.25} + K^{0.25})^3$$

   (a) What is the formula for MRTS?

   (b) Does this production function exhibit decreasing, constant, or increasing returns to scale?

   (c) What is the elasticity of substitution?

2. Do problem 11.6 parts a and b from the Nicholson reading.

3. *VisaDiscover.* Visa and Discover are considering the introduction of debit cards. Both firms have the same production function $f(L,K) = L^{.8}K^{.3}$. Labor and capital both cost \$10 per unit.

   (a) What is the long run total cost curve for either company? Use the Lagrangian to show your answer.

   (b) Assume K is fixed in the short run. Confirm that the short-run total cost curve is $TC(y|K) = 10K + 10K^{-0.375}y^{1.25}$.

4. *OilRefineries.* A common argument against environmental regulations is that they will act like a tax and raise the price of goods. When there is a lumpiness to costs, however, this may turn out not to be true. For example, in 1990 several amendments were passed to the Clean Air Act which required oil refineries to significantly upgrade their capital. The surprising result was that the refining industry increased its total supply and the market prices of refined products actually fell, *ceteris paribus*. Here's how this can happen.

Suppose the demand for oil in the U.S. is 15.4 million barrels per day, and is perfectly inelastic. Suppose we can treat all U.S. oil refineries as a single firm with production function

$$f(L, K) = 0.147L^{0.3}K^{0.6}$$

Let $w = 10$, but keep r as a parameter.

(a) Use the Lagrangian to find the conditional factor demands for labor and capital.

(b) Suppose that in 1989, the refining industry had $K = 359$ and was stuck in the short-run. Show that the short-run marginal cost curve for the whole industry is

$$MC(y|K = 359) = 0.153y^{2.33}$$

(c) During the early 1990s, oil refineries did not move into the long run because there were additional costs associated with shutting down the refineries in order to replace the capital. If the firms remain stuck in the short run, what would be the price of oil assuming perfectly competitive behavior? What is the firm's average total cost if $r = 1.$?

(d) Now suppose that the government regulation raises the cost of capital to $r = 1.15$ and it forces the firms to move to the long run so they can adjust their capital. If demand is still 15.4, what would be the price of oil? Now what would be the firm's average total cost?

(e) Graph the supply-demand equilibrium of parts (c) and (d).

(f) For each part above, a-d, what, in your opinion, is the most limiting assumption underlying the choice of modeling approach. For example for part (a), your answer might be "It is not realistic to treat all refineries as a single firm because." (This example is not a very good answer, however.) For each limitation, say what is being being oversimplified, and how addressing the concern might be expected to change the answer.

Review Problems, not to turn in:

5. *Obesity.* Suppose that utility for an individual's food consumption (c) and the *average* American's food consumption (x) is given by

$$U(c,x) = \left(\frac{c^{1/2} - \frac{1}{2}x^{1/2}}{\frac{1}{2}}\right)^{3/2} - \frac{c}{w}$$

Assume that $c > 0.25x$, and note that w is a parameter representing the wage (i.e. how hard you must work for your food).

   (a) What is the first order condition for optimal c, assuming an individual just takes x as given?

   (b) If the wage rises (w goes up), how does this change the first order condition? What happens to consumers' food consumption?

   (c) Write down the maximization problem and first order condition for a social planner. At the symmetric solution, where $c = x$, does the planner give people as much food to consume as they would choose by themselves? Why or why not?

6. Do problem 11.1 from the Nicholson reading.

7. *Tequila.* The spirit tequila is produced by distilling the fermented juice of the agave plant. True tequila can only be made from agave grown in the officially denominated tequila region in the environs of Tequila, Mexico. An agave plant takes 8 years to reach maturity, and the region was hit by a freak frost in 1997 that killed many agave plants.

Suppose that tequila is produced according to the following production function:

$$f(x,a) = 143x^{0.05}a$$

a is metric tons of agave and x is a composite factor including labor, oak casks, grinding equipment, and so forth. The idea behind this production function is that if $x = 1$, each ton of agave produces 143 liters of tequila, but x could be adjusted to change this amount.

Let the price of X be 400 pesos and the price of a metric ton of agave is 1,000 pesos.

(a) What is the long-run cost curve for tequila?

(b) If distillers set LRAC(q)=6, how much a do they use?

(c) Suppose that after the freeze, 90% of the amount of a from part (b) remains available, and so it becomes a fixed factor. If, nevertheless, distillers want to produce the same ouput, what is the cost?

8. *Technologies*. Suppose there are three technologies for providing a new Internet service, and one of them will eventually emerge as clearly superior to the other two. All three technologies use the factors S for servers and B for bandwidth, and the factor prices are $w_S = w_B = 1$.

   Technology A has production function $F(S,B) = S^{0.7}B^{0.7}$ but S must be set to 10 units and can never be changed.

   Technology B has production function $F(S,B) = S^{0.6}B^{0.6}$ and both factors can be freely varied.

   Technology C has production function $F(S,B) = S^{0.3}B^{0.6}$ and both factors can be freely varied.

   Which technologies have economies of scale? Which have diseconomies of scale?

Answer to Review Problems:

5. *Obesity_a.*

   (a) The first order condition is:

$$\frac{\partial U(c,x)}{\partial c} = \frac{3}{2}\left(\frac{c^{1/2} - \frac{1}{2}x^{1/2}}{\frac{1}{2}c}\right)^{1/2} - \frac{1}{w} = 0$$

   (b) If the wage rises, the first order condition shifts up and to the right. Consumption must rise to preserve the equality.

   (c) The planner will choose everyone's c all at once, thus solving:

$$\max_c U(c,c) = \left(\frac{c^{1/2} - \frac{1}{2}c^{1/2}}{\frac{1}{2}}\right)^{3/2} - \frac{c}{w} = c^{3/4} - \frac{c}{w}$$

   The planner's first order condition is

$$\frac{dU(c,c)}{dc} = \frac{3}{4}c^{-1/4} - \frac{1}{w} = 0$$

   If we evaluate the individual's FOC at $c = x$, it simplifies to

$$\left.\frac{\partial U(c,x)}{\partial c}\right|_{c=x} = \frac{3}{2}\left(\frac{c^{1/2} - \frac{1}{2}c^{1/2}}{\frac{1}{2}c}\right)^{1/2} - \frac{1}{w} = \frac{3}{2}c^{-1/4} - \frac{1}{w} = 0$$

   The planner gives less food to the consumers because the planner takes into account the disutility that results from overall food consumption, presumably through increased health problems.

6. *Nicholson11.1_a*

   (a) The graph is upward sloping and concave.

   (b) $AP_L = \frac{100\sqrt{L}}{L} = 100L^{-0.5}$. This is clearly decreasing in L.

   (c) $\frac{dq}{dL} = 100 \cdot 0.5L^{-0.5} = \frac{50}{\sqrt{L}}$. Because the total product of L is concave everywhere, it is always the case that MP_L is diminishing. That means that each additional worker always pulls down the average product.

7. *Tequila_a.*

(a) Let's use the MRTS to find the answer:

$$\text{MRTS} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial a}} = -\frac{0.05 \cdot 143 a x^{-0.095}}{143 x^{0.05}} = -0.05\frac{a}{x}$$
$$-0.05\frac{a}{x} = -\frac{400}{1000} \Rightarrow \frac{a}{x} = 8 \Rightarrow a = 8x$$

Then $q = 143(8x)\cdot x^{0.05} = 1144x^{1.05}$ and inverting this we find that to produce q, the conditional factor demand is

$$x(q) = 0.0012q^{0.95}$$

Thus the cost of q is:

$$TC(q) = 400 \cdot 0.0012q^{0.95} + 1000 \cdot 8 \cdot 0.0012q^{0.95} = 10.27q^{0.95}$$

(b) The long run average cost is

$$AC(q) = \frac{TC(q)}{q} = 10.27q^{-0.05}$$

If $AC(q) = 6$, then

$$10.27q^{-0.05} = 6 \Rightarrow q = 46433$$

Now we use the conditional factor demand:

$$x(46433) = 32.56 \qquad a(46433) = 260.5$$

(c) The short-run problem is:

$$f(0.9 \cdot 260.5, x) = 143 \cdot 234.45 \cdot x^{0.05}$$
$$\Rightarrow q = 33526.35x^{0.05}$$

To produce output 46433, the producers need:

$$46433 = 33526.35x^{0.05} \Rightarrow x = 674.23$$

Thus, the new total cost is

$TC(46433|a = 234.45) = 1000{\cdot}234.45 + 400{\cdot}674.23 = 504142.82$

Thus, the freeze caused a huge cost increase, assuming production did not change.

8. *Technologies_a*. Technology B always has economies of scale at all output levels because the exponents add to more than 1.

   Technology C has exponents that add to less than 1, so there are diseconomies of scale at all levels of output.

   For Technology A, we can rewrite the production function as $F(B) = 5B^{0.7}$. Thus, $B = 0.1y^{10/7}$ and $c(y) = 10 + 0.1y^{10/7}$. The average cost is $AC(y) = \frac{10}{y} + 0.1y^{3/7}$. The marginal cost is $MC(y) = \frac{1}{7}y^{3/7}$. To find the bottom of the U of average cost

$$\begin{aligned} AC(y) &= MC(y) \\ \frac{10}{y} + 0.1y^{3/7} &= \frac{1}{7}y^{3/7} \\ \frac{10}{y} &= 0.04y^{3/7} \\ 10 &= 0.043y^{10/7} \\ y &= 45.34 \end{aligned}$$

   So for $y < 45.34$ there are economies of scale, but for greater y there are diseconomies.