

ECON 301, Professor Hogendorn

Problem Set 6

1. *LRSR*. A firm has the following production function:

$$f(K, L) = \sqrt{K} + \sqrt{L}$$

where K is capital and L is labor. The price of K is 1 and the price of L is 2.

- (a) Write the Lagrangian and find the first order conditions.
- (b) Use Mathematica to solve simultaneously and confirm that the long-run conditional factor demands are

$$K(y) = \frac{4y^2}{9}$$

$$L(y) = \frac{y^2}{9}$$

- (c) Use Mathematica to find the LRTC, LRMC, and LRAC.
2. *Luxray*. Luxray Inc. is a firm with cost function $TC(y) = y^2 + 10$. This firm is a perfectly competitive price-taker in a market where $p = 100$.

- (a) Write down Luxray's average cost function, average variable cost function, and marginal cost function. Why does Luxray produce $y^* = 50$ in short-run equilibrium?
- (b) Find Luxray's net profit at $y^* = 50$. Write it down three ways and verify that they are all equal: (i) total revenue minus total cost, (ii) net profit margin times quantity, (iii) operating profit margin times quantity, minus fixed costs.

(c) Suppose that the cost functions we have been working with *are* the long-run cost functions. However, firms may enter or exit this market freely in the long run. Should Luxray management expect to produce more or less than 50 units as the industry moves toward long-run equilibrium? Explain using a graph.

3. *OilProducers.* Suppose there are two oil-producing regions in the world, and in each one there are perfectly competitive producers. The fixed cost is F and the long-run average variable costs are

$$\text{LAVC}(y) = y$$

(FYI, output is measured in thousand barrels per day, MBbl/d.) For an oil shale deposit, $F = \$3,000,000$. In Saudi Arabia, the capital cost is so much lower, we might as well just set $F = 0$.

- (a) Show cost-curve diagram for a Saudi oil well and an oil shale well. Draw the LAC (i.e. long-run average total cost) curve, the LMC (long-run marginal cost curve), and show a price of \$95 per barrel of oil. What is the optimal output for each type of well?
- (b) Suppose there were N wells of each type. What is the market supply curve for oil (holding the number of wells fixed)?
- (c) Suppose new wells can enter the market, but only by using oil shale. What is the long-run market supply curve for oil?

Review problems only, not to turn in:

4. *12firms.* There is a firm with production function

$$q = f(L, K) = L^{1/2} + K^{1/2}$$

This firm is initially stuck in the short run with $\bar{K} = 16$ which cannot be changed. The wage is $w = 3$ and the price of capital is $r = 4$.

- (a) Find the short run marginal cost curve and the short-run supply curve.
 - (b) If there are 12 firms, and if market demand is $q(p) = 96 - p$, what is the short-run market equilibrium price?
 - (c) What is the short-run average total cost? Is this firm making a loss, breaking even, or making a super-normal profit? Illustrate on a two-panel graph, one panel showing the market, the other showing the cost curves of an individual firm.
5. *Consulting.* Technology can improve labor productivity. One might be concerned that this could be bad for workers since fewer would be needed to produce the same output. Displaced workers might have to move to another industry. To think about this, suppose an industry has the production function $f(L) = \alpha L^{0.5}$. The conditional factor demand is thus $L(y) = \left(\frac{1}{\alpha}\right)^2 y^2$. Let $w_L = 1$ throughout this whole problem (i.e. overall labor market equilibrium is unaffected by the changes in the industry we examine here). Suppose there is a fixed cost to start a firm which is $F = 2500$. The cost function is thus

$$c(y) = \left(\frac{1}{\alpha}\right)^2 y^2 + 2500$$

Note that this is both the short-run and the long-run cost function; the only difference is that in the long run a firm can exit or enter the industry.

Suppose that demand in the industry is given by $X(p) = 60000p^\epsilon$. Elasticity ϵ can take on two values: -0.5 and -1.5. Answer the following for each of these values:

- (a) Suppose that initially $\alpha = 100$ and the industry is in long-run perfectly competitive equilibrium. How many firms are there? What is the total number of workers?

- (b) Suppose that improved technology causes a change to $\alpha = 160$. In the short run (i.e. with the number of firms fixed) what is the new total number of workers?
- (c) In the long run, the number of firms will adjust to the new situation. What is the new number of firms and the new total number of workers?
- (d) Describe in words what an individual worker would experience during parts (a)-(c). For example, your description might read “First I noticed that my firm hired a few new people. Later, some of our competitors went out of business. Most of the people who worked for those firms came to work at my firm and our remaining competitors, but a few had to get jobs in another industry.” Remember to do this for both values of elasticity, and discuss which elasticity is preferable for the workers.

Answers to Review Problems:

4. *12firms_a.*

- (a) Since $q = L^{1/2} + 4$, the short-run conditional factor demand for labor is

$$L^{1/2} = q - 4 \Rightarrow L(q) = (q - 4)^2$$

Short run total cost $TC(q) = wL(q) + r\bar{K} = 3(q-4)^2 + 64$. Then marginal cost is

$$MC(q) = \frac{dTC(q)}{dq} = 6(q - 4) = 6q - 24$$

A perfectly competitive firm sets $MC=p$, so its supply is

$$p = 6q - 24 \Rightarrow q = \frac{p + 24}{6} \Rightarrow s(p) = 4 + \frac{1}{6}p$$

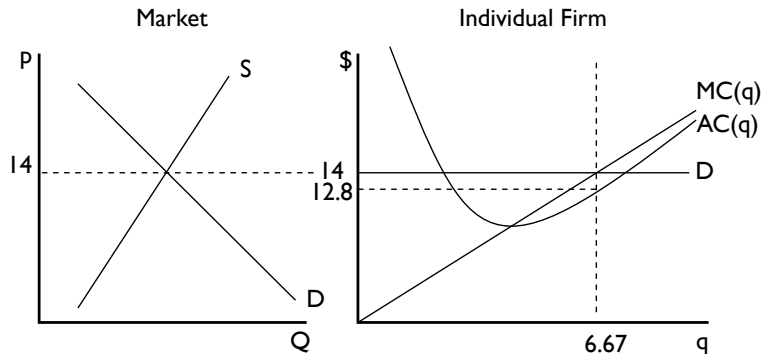
(b) With 12 firms, market supply equals market demand is

$$12s(p) = q(p) \Rightarrow 48 + 2p = 96 - p \Rightarrow p^* = 16$$

(c) At $p = 16$, each individual firm produces $s(16) = 6.67$. Short run average cost is

$$AC(q) = \frac{TC(q)}{q} = \frac{3(q - 4^2)}{q} + \frac{64}{q}$$

so $AC(6.67) = 3.2 + 9.6 = 12.8$. Since this is lower than the price of 16, the firm makes a profit.



5. Consulting_a.

(a) The average and marginal cost curves in this case are:

$$AC(y) = \frac{2500}{y} + 0.0001y \quad MC(y) = 0.0002y$$

Thus, the minimum average cost is $\frac{2500}{y} + 0.0001y = 0.0002y \Rightarrow y_{LR} = 5000$. At this output, the amount of labor employed by each firm is $L(5000) = 2500$.

The marginal cost of this output level is $MC(5000) = 1$, and since perfectly competitive firms set price equal to marginal cost, we have $p_{LR} = 1$. This is the long run supply curve.

Equating supply to demand, we find the demand at $p = 1$, which is $60000 \cdot 1^e = 60000$.

The number of firms in the market must therefore be $N = \frac{60000}{5000} = 12$. Since each firm employs 2500 workers, total employment is 30000.

- (b) Now that $\alpha = 160$, the conditional factor demand is $L(y) = 0.00004y^2$ and the total cost function is $c(y) = 0.00004y^2 + 2500$. Thus, the new marginal cost curve and the short-run firm supply curve is:

$$MC(y) = 0.00008y \quad s(p) = 12500p$$

Since the number of firms cannot change in the short run, there are still 12 of them, so the market supply curve is just $12s(p)$, and setting supply equal to demand gives us:

$$60000p^\epsilon = 12 \cdot 12500p$$

$$p^{\epsilon-1} = 2.5$$

$$p = 2.5^{\frac{1}{\epsilon-1}}$$

$$p = (0.54, 0.69) \text{ when } \epsilon = (-0.5, -1.5)$$

$$X(p) = (81650, 104683) \text{ when } \epsilon = (-0.5, -1.5)$$

$$y = (6804, 8724) \text{ when } \epsilon = (-0.5, -1.5)$$

$$L(y) = (1852, 3044) \text{ when } \epsilon = (-0.5, -1.5)$$

$$NL(y) = (22224, 36528) \text{ when } \epsilon = (-0.5, -1.5)$$

- (c) With $\alpha = 160$, the average and marginal cost curves are:

$$AC(y) = \frac{2500}{y} + 0.00004y \quad MC(y) = 0.00008y$$

Thus, the minimum average cost is $\frac{2500}{y} + 0.00004y = 0.00008y \Rightarrow y_{LR} = 7906$. At this output, the amount of labor employed by each firm is $L(7906) = 2500$. (Note this is the same as before, which occurs because we have only changed the coefficient on the production function.)

The marginal cost of this output level is $MC(7906) = 0.632$, and since perfectly competitive firms set price equal to marginal cost, we have $p_{LR} = 0.632$. This is the long run supply curve. Equating supply to demand, we find:

$$p = (0.632, 0.632) \text{ when } \epsilon = (-0.5, -1.5)$$

$$X(p) = (75473, 119420) \text{ when } \epsilon = (-0.5, -1.5)$$

$$N = (9.54, 15.1) \text{ when } \epsilon = (-0.5, -1.5)$$

$$L(y) = (2500, 2500) \text{ when } \epsilon = (-0.5, -1.5)$$

$$NL(y) = (23866, 37750) \text{ when } \epsilon = (-0.5, -1.5)$$

- (d) Case of $\epsilon = -0.5$: I never should have taken the job at Sprint. Everything was fine until stupid researchers at Bell Labs and Nortel introduced the new technology. There was overcapacity everywhere, and Sprint laid off about 25% of its workforce. After a while, Global Crossing, Williams, and Worldcom filed for bankruptcy. But now that there's been some consolidation, Sprint is doing a little better, and it looks like the laid-off workers will be rehired. My friends at Global Crossing are out of luck though – there won't be any telecoms jobs for them.
- Case of $\epsilon = -1.5$: When I started at Nortel, it seemed like a sleepy firm, but then this great new technology came along. Nortel grew really fast, and we hired all kinds of new people. Fortunately, I saw that the good times couldn't last, so I cashed in my stock options and moved to a startup. It's a good thing, because Nortel laid off most of the people it hired. My new firm's hanging in there, but it's not like before.
- Clearly, the $\epsilon = -1.5$ is preferable, but note that even then there were some layoffs in this model.