ECON 301, Professor Hogendorn

Problem Set 9

1. *SubsInc.* Let your utility function be

$$u(x, y) = \sqrt{x} + \sqrt{y}$$

   This will give demand functions

$$x(p_x, p_y, m) = \frac{m}{\frac{p_x^2}{p_y} + p_x} \qquad y(p_x, p_y, m) = \frac{m}{\frac{p_y^2}{p_x} + p_y}$$

   (You can try this for a review of the Lagrangian for the final.)

   If $p_x = 1$, $m = 100,000$, and p_y starts at \$1 and then rises, what is the substitution effect and the income effect?

2. *BigMacs.* You buy a lot of Big Macs. You are also on your town's zoning board, and McDonald's REALLY wants to build a new restaurant there. McDonald's raises the price of Big Macs from \$3 to \$4. Your demand for Big Macs is $x(p, m) = 0.01 \frac{m}{p^{1.5}}$. Your income m is \$50,000.

   You complain about the price increase, and subtly hint that it could affect your zoning decision. In response, McDonald's sends a representative who will compensate you with coupons for free Big Macs (fractional coupons are allowed). Here are three possible ways to compensate you:

   (a) Calculate a Laspeyres price index, calculate the additional income you would need according to the price index, and divide that amount by \$4 to get the number of Big Mac coupons.

   (b) Use Slutsky income-compensated demand to calculate the substitution effect, and give that many Big Mac coupons.

(c) Use Marshallian demand to calculate the change in demand, and give that many Big Mac coupons.

3. *HouseBuying.* First, recall that for any good x for which a consumer has endowment ω_x, the Slutsky equation can be written

$$\frac{\partial x}{\partial p_x} = \frac{dx^s}{dp_x} + (\omega_x - x^*)\frac{\partial x}{\partial m}$$

Suppose you want to use the Slutsky equation to examine the effect of a housing price increase on two types of households who buy new houses. In this problem, houses are measued in units of h, where a higher h signifies a bigger, fancier, better-located house and lower h signifies a smaller, simpler, worse-located house. House prices are quoted in p_h per unit of h.

(a) A move-up household owns (is endowed with) a house of type ω_h and buys a bigger house of type $h^* > \omega_h$. Write the Slutsky equation for the move-up household. Discuss the sign of each term.

(b) Draw an indifference curve/budget line diagram of the move-up household showing the endowment point, the initial purchase point h^*, and the effect of an increase in house prices p_h. Label the substituion and income effects.

(c) Now consider an older couple who want to downsize. They are endowed with a house of type ω'_h and want to buy a smaller house of type $h^* < \omega'_h$. Use the Slutsky equation to determine how a house-price increase would affect the size of the downsizing couple's new house. (Remember that even though they want a smaller house, h also measures the fanciness of the house so larger h is still a good thing for them all other things equal.)

(d) Actually, you decide that all of the above calculations are making your study very complicated. What you would like to do

is just insert a sentence saying "In this study we ignore income effects," and then just continue with a simpler analysis. Based on the Slutsky equation, what are three characteristics of the households' demands that would help justify ignoring income effects? Make sure to explain each intuitively, not just mathematically.

4. *Relax*. The demand for relaxation is

$$R(w, p, m) = \frac{1}{4}m + p + \frac{1}{w}$$

w is the wage. p is the price of consumption. There are 16 total hours available, and nonlabor income is 12, so total income is $m = 16w + 12$.

   (a) What is the labor supply curve? Is it backward-bending?

   (b) Denoting (C^*, R^*) as the initial consumption bundle, write the Slutsky equation for relaxation.

   (c) Evaluate the Slutsky equation at $p = 1$, $w = 0.6$.

   (d) With reference to the income and substitution effects, explain why labor supply curves often bend backward.

Review problems only, not to turn in:

5. *MrLee*. Mr. Lee is an eccentric millionaire who made his money by manipulating the price of rice in Singapore. He now lives in Middletown, CT, where he purchased a defunct Bradlees department store and converted it to a house. In front of the house is a very large parking lot. Mr. Lee likes to consume large numbers of cars to fill up this parking lot (they can only be the latest model year, so he needs to buy a lot of new cars every year).

Last year the price of Hyundais was $8,000 and the price of Mercedes was $45,000. Mr. Lee bought 200 Hyundais and 25 Mercedes.

These have now been towed away, and it is time to buy this year's cars. Unfortunately, the price of Hyundais has risen to $13,000 this year.

The slope of Mr. Lee's Slutsky compensated demand function for Hyundais is -0.001 (i.e. one less Hyundai for each $1,000 increase in price). The slope of his Engel curve for Hyundais is –0.00001 (i.e. one less Hyundai for each $100,000 increase in income).

(a) Using the Slutsky equation, what is the slope of Mr. Lee's Marshallian demand for Hyundais? How many does he buy this year (assuming the linear estimate of slope can be used)?

(b) Assuming Mr. Lee's income did not change and he spends it all on Hyundais and Mercedes, how many Mercedes does he buy this year?

(c) Graph Mr. Lee's consumption decisions in the two years using budget lines and indifference curves.

(d) Which ones of the following describe Hyundais: normal good, inferior good, Giffen good?

6. *Aisha.* Aisha runs a one-person, ten-cow dairy operation which produces 600 gallons of milk a week. This is her sole source of income. Aisha's utility function is

$$U(x, g) = 60x^2 g^4$$

where x = numeraire and g = gallons of milk. Let p_g be the price of milk.

(a) What is Aisha's demand function for milk?

(b) Show whether milk is a normal or an inferior good.

(c) The price of milk is $4 per gallon. How many gallons of milk does Aisha consume? How much numeraire?

(d) All the dairies except Aisha's are hit by a tornado, wiping out many cows and causing the price of milk to rise. Break down the corresponding change in Aisha's consumption of milk between the substitution, ordinary income, and endowment income effects.

7. *Nurses.* Your state is experiencing a nursing shortage and you, as the state nursing czar, are supposed to figure out how to fix the problem. You don't know the specific function, but you know the labor supply of a typical nurse must be some $L^s(w, m)$, and thus the leisure demand function is $R(w, m)$, where w is the wage and m is the "full income." There is no non-labor income.

Nurses cannot supply more than 10 hours of labor per day due to strict regulation in your state, so the endowment of leisure must be $\overline{R} = 10$. Currently the wage is 20, and currently nurses take R^* hours of leisure and C^* worth of consumption.

(a) Suppose you recommended subsidies that raised the wage to 25. What would be the Marshallian demand for leisure at this new wage? What would be the Slutsky compensated demand for leisure at this new wage?

(b) Will the nurses definitely work more hours at the new wage? Why or why not?

(c) Another option would be to give the nurses a lump sum bonus of $75 per day. What would be the Marshallian and Slutsky compensated demands for leisure under this option?

(d) Would this work better or worse than the wage increase at alleviating the nursing shortage?

Answer to Review Problems:

5. *MrLee_a.* Let h be Hyundais, d be Mercedes, and m be income.

(a) Substituting into the Slutsky equation gives us:

$$\frac{\partial h(p_h, p_d, m)}{\partial p_h} = \frac{\partial h^s}{\partial p_h} - \frac{\partial h(p_h, p_d, m)}{m} h^*$$

$$\frac{\partial h(p_h, p_d, m)}{\partial p_h} = -0.001 - (-0.00001)200$$

$$= +0.001$$

To estimate the number purchased this year:

$$\frac{\partial h(p_h, p_d, m)}{\partial p_h} \cdot \$5000 = 5$$

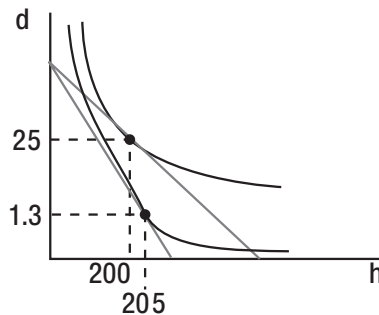so 205 Hyundais this year.

(b) Last year's income must have been

$$m = 8000 \cdot 200 + 45000 \cdot 25 = 2725000$$

This year's budget constraint is:

$$13000 \cdot 205 + 45000 \cdot d^* = 2725000 \Rightarrow d^* = 1.3$$

(c) The graph is:

(d) Hyundais are an inferior good because their Engel curve slopes down, and they are a Giffen good because the Marshallian demand curve slopes up due to the very strong income effect of a price change.

6. *Aisha_a.*

   (a) Since the utility function is Cobb-Douglas, we know that the demand function will take the form $g(p_g, m) = \frac{2}{3}\frac{m}{p_g}$ where m is the *full income.*

   Note that the Cobb-Douglas form means that Aisha always spends 2/3 of her income on milk.

   In this case, $m = 600p_g$, so $g(p_g, 600p_g) = \frac{2}{3}600 = 400$.

   Since Aisha's income depends only on the price of milk, it turns out that her milk consumption is constant. This unusual result occurs because the endowment income effect will completely cancel the ordinary income and substitution effects. It would not occur if, for example, Aisha had an endowment of x as well.

   (b) Demand for a normal good increases when income increases. Here,
   $$\frac{\partial g}{\partial m} = \frac{2}{3p_g} > 0$$
   so milk is normal. Note that what we want here is the slope of the Engel curve, which is a *partial* derivative in which m increases but p_g stays constant.

   (c) We already saw that $g(p_g, 600p_g) = 400$ for any p_g. If $p_g = 4$, $m = 600 Ö 4 = 2400$. We know that Aisha always spends 2/3 of her income on milk and 1/3 on other goods, so 2400/3=800 is the amount spent on x. And since $p_x = 1$, x=800.

   (d) The derivative of Slutsky compensated demand is
   $$\frac{\partial g^s}{\partial p_g} = \frac{\partial g}{\partial p_g} + \frac{\partial g}{\partial m}g^*$$
   $$\frac{\partial g^s}{\partial p_g} = -\frac{2}{3}\frac{m}{p_g^2} + \frac{2}{3p_g}400$$

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$$\frac{\partial g^s}{\partial p_g} = -\frac{2}{3}\frac{2400}{4^2} + \frac{2}{3\ddot{O}4}400$$

$$\frac{\partial g^s}{\partial p_g} = -100 + 67 = -33$$

Since Aisha has an endowment of g=600, the total derivative of her Marshallian demand function is

$$\frac{dg}{dp_g} = \frac{\partial g}{\partial p_g} + \frac{\partial g}{\partial m}600 = 0$$

(recall we found this was equal to 0 in part a.) Combining this with the Slutsky compensated demand gives

$$\frac{dg}{dp_g} = \frac{\partial g^s}{\partial p_g} - \frac{\partial g}{\partial m}g^* + \frac{\partial g}{\partial m}600$$

Filling in from above we find

$$0 = -33 - 67 + \frac{2}{3\ddot{O}4}600 = -33 - 67 + 100$$

Thus the substitution effect is -33, the ordinary income effect is -67, and the endowment income effect exactly offsets these at $+100$.

7. *Nurses_a.*

   (a) The new Marshallian demand is $R(25, 250)$. Note how both the price and the income have risen; this is why we need the Slutsky equation to separate out what is happening. The Slutsky compensated demand at the new wage is

   $$R^s(25) = R(25, 25R^* + C^*)$$

   (b) Because this is an endowment problem, we start by taking the derivative of Marshallian demand.

   $$\frac{dR}{dw} = \frac{\partial R}{\partial w} + \frac{\partial R}{\partial m}10$$

Next, we take the derivative of Slutsky compensated demand and rearrange it:

$$\frac{\partial R^s}{\partial w} = \frac{\partial R}{\partial w} + \frac{\partial R}{\partial m} R^*$$

$$\frac{\partial R}{\partial w} = \frac{\partial R^s}{\partial w} - \frac{\partial R}{\partial m} R^*$$

Finally, we can substitute the last equation into the derivative of Marshallian demand to get the three effects:

$$\frac{dR}{dw} = \frac{\partial R^s}{\partial w} - \frac{\partial R}{\partial m} R^* + \frac{\partial R}{\partial m} 10$$

On the right hand side, the first term *must* be negative since it is a compensated demand curve. The derivative $\frac{\partial R}{\partial m}$ is the slope of the Engel curve for leisure – one would assume that it is positive. Thus, the last two terms, the ordinary plus endowment income effect will total up to something positive multiplied by $10 - R^*$, the amount of work the nurses choose to do.

We cannot be sure whether the nurses will work more hours, but we can say that they are less likely to work more if (i) they regard leisure as more of a luxury good and (ii) they already were working most of the possible hours (R^* close to 0).

(c) The new Marshallian demand would just add the \$75 to income: $R(w, 10w+75)$. The new Slutsky compensated demand would compensate to $R(w, wR^* + C^* + 75)$.

(d) Since this is a pure income change, we only need to know the slope of the Engel curve. The effect of the change would be:

$$\frac{\partial R}{\partial m} 75$$

Since leisure is a normal good, this just makes it even more likely that they will take additional leisure. So the bonus is a terrible idea!