ECON 321, Prof. Hogendorn

Problem Set 1 Answers

1. *M2-1_a*. The monopoly profit maximization problem is:

$$\max_q \pi(q) = (100 - q - 10)q$$

The first order condition (equivalent to setting marginal revenue equal to marginal cost) is

$$\frac{\partial \pi}{\partial q} = 90 - q - q = 0 \Rightarrow q = 45$$

so the price is $p = 100 - 45 = 55$. The Lerner Index is

$$\frac{p - MC}{p} = \frac{55 - 10}{55} = 0.82$$

which is to say that 82% of the price is monopoly markup.

Under perfect competition, the price would be 10, and quantity would be $10 = 100 - q \Rightarrow q = 90$. Thus, the deadweight loss is a triangle with base of $90 - 45$ and a height of $55 - 10$, which has area:

$$DWL = \frac{1}{2}(90 - 45)(55 - 10) = 1012.5$$

2. *KmartWalMart_a*. In this case, Wal-Mart has $MC_W = 0.7$ while Kmart has $MC_K = 1$. So to begin, we find the Cournot reaction function for a firm 1 with marginal cost c facing a rival firm 2:

$$\max_{q_1} = (2 - 4(q_1 + q_2))q_1 - cq_1$$

$$\begin{aligned} \text{FOC:} \quad 2 - 4q_1 - 4q_2 - 4q_1 &= c \\ 2 - 8q_1 - 4q_2 &= c \\ 8q_1 &= 2 - c - 4q_2 \\ q_1(q_2) &= 0.25 - \frac{c}{8} - \frac{q_2}{2} \end{aligned}$$

Note the above is true for any type of firm, since firms don't *directly* care about opponents' costs, but only indirectly through the effect on q_2. If we now plug in the marginal costs given in the problem, we get

$$q_K(q_W) = 0.125 - \frac{q_W}{2} \qquad q_W(q_K) = 0.1625 - \frac{q_K}{2}$$

Now solving simultaneously to find a Cournot-Nash equilibrium:

$$\begin{aligned} q_K &= 0.125 - \frac{0.1625 - \frac{q_K}{2}}{2} \\ q_K &= 0.125 - 0.08125 + \frac{q_K}{4} \\ q_K\left(1 - \frac{1}{4}\right) &= 0.04375 \\ q_K &= 0.058 \end{aligned}$$

And plugging this back into the q_W function gives

$$q_W = 0.1625 - \frac{0.058333}{2} = 0.13$$

At these outputs, the price is $p(0.13 + 0.058) = 1.23$, Kmart's profit is $(1.23 - 1)0.058333 = 0.0136$ and Wal-Mart's profit is $(1.23 - 0.7)0.13 = 0.0689$. The market shares are $0.058/0.188 = 30.8\%$ for Kmart and $0.13/0.188 = 69\%$ for Wal-Mart.