ECON 321, Professor Hogendorn

Problem Set 3

1. *Chocolate.* Two firms supply cacao at a wholesale market in Trinidad. Firm 1 has always had lower costs than firm 2, reflected by constant marginal costs $c_1 < c_2$. Market demand for cocoa is $p = a - Q$, where $Q = q_1 + q_2$.

   (a) If these firms behaved like perfect competitors, what would be each firm's output, market price, and market quantity?

   (b) Now suppose the two firms behave as Cournot competitors. What is the Cournot equilibrium quantity produced by both firms, the market price, and the equilibrium profit of firm 1?

   (c) If firm 1 could move first, followed by firm 2, what would be the Stackelberg equilibrium quantity produced by both firms, the market price, and the equilibrium profit of firm 1?

2. *AccBert.* Consider a two-stage game with 2 firms. In Stage 1, firm 1 can buy a machine at fixed cost 0.5. The machine lowers its marginal cost to 0. Alternatively, firm 1 can not buy the machine, in which case its marginal cost is 1.

   In stage 2 of the game, the two firms compete à la differentiated Bertrand with the demand system used in the review problem BertrandCollusion (see below). This system is:

   $$q_1 = 1 - 0.3p_1 + 0.1p_2 \qquad q_2 = 1 - 0.3p_2 + 0.1p_1$$

   Firm 2 has marginal cost of 1 no matter what.

   (a) What are the stage 2 reaction functions and profit outcome if firm 1 does not buy the machine? (This is easy, it's the same answer as part (a) of the BertrandCollusion problem, but make sure you understand.)

(b) What are the period 2 reaction functions and profit outcome if firm 1 does buy the machine?

(c) What is the stage 1 equilibrium: firm 1 does buy or does not buy the machine?

Review problems:

3. *BertrandCollusion.* There are two firms which are differentiated Bertrand competitors. They have demand curves:

$$q_1 = 1 - 0.3p_1 + 0.1p_2 \qquad\qquad q_2 = 1 - 0.3p_2 + 0.1p_1$$

The firms have identical, constant marginal costs of \$1 per unit.

(a) What is the differentiated Bertrand equilibrium profit?

(b) If the two firms colluded, what would be the profit of each firm? (You can confine attention to actions that set the prices equal since the firms are symmetric.)

(c) If one firm cheated on this collusive agreement, what profit would it make?

(d) Suppose the game were repeated. For what discount rates could the firms sustain a tacitly collusive trigger strategy equilibrium?

4. *Shy6.8.3.* Consider a 3-firm version of the Stackelberg game. Assume that market inverse demand is given by $p = 120 - Q$, and suppose that there are three firms that set their output sequentially: firm 1 sets q_1 in period 1, firm 2 sets q_2 in period 2, and firm 3 sets q_3 in period 3. Then, firms sell their output and collect their profits.Solve for the sequential-moves (i.e. Stackelberg) equilibrium. Make sure that you solve for the output level of each firm, and for the market price.

Answers to review problems:

3. *BertrandCollusion_a.*

   (a) Firm 1's profit maximization problem is:

   $$\max_{p_1} \pi_1(p_1, p_2) = (p_1 - 1)(1 - 0.3p_1 + 0.1p_2)$$

   Its first order condition is:

   $$\frac{\partial \pi}{\partial p_1} = 1.3 - 0.6p_1 + 0.1p_2 = 0$$

   Solving for p_1 and exploiting the symmetry of the problem, we can get the reaction functions for firms 1 and 2:

   $$p_1(p_2) = \frac{1.3 + 0.1p_2}{0.6} \qquad p_2(p_1) = \frac{1.3 + 0.1p_1}{0.6}$$

   When we set these equal and solve simultaneously, the Nash equilibrium is $p_1^* = p_2^* = 2.6$. The corresponding quantities are
   $q_1(2.6, 2.6) = q_2(2.6, 2.6) = 0.48$. and the profits are
   $\pi_1(2.6, 2.6) = \pi_2(2.6, 2.6) = 0.768$.

   (b) The problem now is to choose one collusive price p^c that maximizes the combined profits of the firms:

   $$\max_{p^c} 2\pi(p^c, p^c) = 2(p^c - 1)(1 - 0.3p^c + 0.1p^c)$$

   The first order condition is

   $$\frac{\partial 2\pi}{\partial p^c} = 2.4 - 0.8p^c = 0 \Rightarrow p^c = 3$$

   At this price, both firms produce quantities of 0.8 and make profits of 0.8.

(c) In this case, firm 1 knows that firm 2 will play by the rules and choose $p_2 = 3$. For firm 1 finds its best response using the curves we derived in (a), and gets

$$p_1(3) = \frac{1.3 + 0.1 \cdot 3}{0.6} = 2.67$$

And at this price it earns profit $\pi(2.67, 3) = 0.835$.

(d) The firms can sustain a trigger strategy equilibrium as long as the endlessly repeated payoff to cooperating is greater than the cheating payoff plus the endlessly repeated Nash game. Letting β be the discount rate, this requires that:

$$\frac{0.8}{1-\beta} \geq 0.835 + \beta \frac{0.768}{1-\beta}$$

This works for any β greater than 0.52.

4. *Shy6.8.3_a.* We have to work backwards, starting with stage 3. At that point, q_1 and q_2 are given, and firm 3 maximizes:

$$\max_{q_3} \pi_3 = (120 - q_1 - q_2 - q_3)q_3$$

This gives first order condition

$$\frac{\partial \pi_3}{\partial q_3} = 120 - q_1 - q_2 - q_3 - q_3 = 0$$

and reaction function

$$q_3(q_1, q_2) = 60 - \tfrac{1}{2}q_1 - \tfrac{1}{2}q_2$$

In Stage 2, firm 2 maximizes its profits, expecting firm 3 to behave as we just derived:

$$\max_{q_2} \pi_2 = (120 - q_1 - q_2 - q_3(q_1, q_2))q_3 = (60 - \tfrac{1}{2}q_1 - \tfrac{1}{2}q_2)q_2$$

This gives first order condition

$$\frac{\partial \pi_2}{\partial q_2} = 60 - \tfrac{1}{2}q_1 - \tfrac{1}{2}q_2 - \tfrac{1}{2}q_2 = 0$$

and reaction function

$$q_2(q_1) = 60 - \tfrac{1}{2}q_1$$

In Stage 1, firm 1 knows that firm 2 will play as above. It also knows that firm 3 will go on to react according to

$$q_3(q_1, q_2(q_1)) = 60 - 0.5q_1 - 30 + 0.25q_1 = 30 - 0.25q_1$$

Thus, firm 1 maximizes

$$\max_{q_2} \pi_2 = (120 - q_1 - (60 - 0.5q_1) - (30 - 0.25q_1))q_1 = (30 - 0.25q_1)q_1$$

This gives first order condition

$$\frac{\partial \pi_1}{\partial q_1} = 30 - 0.25q_1 - 0.25q_1 = 0$$

and optimal Stackelberg leader quantity

$$q_1 = 60$$

Plugging this leader quantity back into the other reaction functions, we see that the other two firms produce

$$q_2 = 30 \qquad q_3 = 15$$

The total market quantity is thus 105, and the market price is $p = 120 - 105 = 15$.