ECON 321, Professor Hogendorn

Problem Set 3 Answers

1. *Chocolate_a.*

   (a) Under perfect competition, firms set marginal cost equal to price. Since firm 2 has a higher cost, it cannot compete in the market at all, so $q_2 = 0$.

   That leaves only firm 1 in the market, and the market price of $p_1 = c_1$. At that price, firm 1's quantity equals market quantity: $q_1 = Q = a - c_1$.

   (b) The firms' profit maximization problems are:

   $$\max_{q_1} \pi_1 = (a - q_1 - q_2)q_1 - c_1 q_1 \qquad \max_{q_2} \pi_2 = (a - q_1 - q_2)q_2 - c_2 q_2$$

   Taking first order conditions for both problems and solving for q_1 and q_2 gives reaction functions:

   $$q_1(q_2) = \frac{a - c_1 - q_2}{2} \qquad q_2(q_1) = \frac{a - c_2 - q_1}{2}$$

   Solving these simultaneously gives the Cournot-Nash equilibrium quantities

   $$q_1^* = \frac{a - 2c_1 + c_2}{3} \qquad q_2^* = \frac{a - 2c_2 + c_1}{3}$$

   At these quantities, the market price and firm 1's profits are:

   $$p^* = \frac{a + c_1 + c_2}{3} \qquad \pi_1^* = \frac{(a - 2c_1 + c_2)^2}{9}$$

   (c) Now firm 1 anticipates firm 2's reaction when it solves its new profit maximization problem:

   $$\max_{q_1} \pi_1 = \left(a - q_1 - \frac{a - c_2 - q_1}{2}\right) q_1 - c_1 q_1$$

Firm 1's solution to this problem, and the corresponding response of firm 2, are:

$$q_1^* = \frac{a - 2c_1 + c_2}{2} \qquad q_2^* = \frac{a - 3c_2 + 2c_1}{4}$$

Then the market price and the profit for firm 1 are

$$p^* = \frac{a + 2c_1 + c_2}{4} \qquad \pi_1^* = \frac{(a - 2c_1 + c_2)^2}{8}$$

2. *AccBert_a.*

   (a) Firm 1's profit maximization problem is:

   $$\max_{p_1} \pi_1(p_1, p_2) = (p_1 - 1)(1 - 0.3p_1 + 0.1p_2)$$

   Its first order condition is:

   $$\frac{\partial \pi}{\partial p_1} = 1.3 - 0.6p_1 + 0.1p_2 = 0$$

   Solving for p_1 and exploiting the symmetry of the problem, we can get the reaction functions for firms 1 and 2:

   $$p_1(p_2) = \frac{1.3 + 0.1p_2}{0.6} \qquad p_2(p_1) = \frac{1.3 + 0.1p_1}{0.6}$$

   When we set these equal and solve simultaneously, the Nash equilibrium is $p_1^* = p_2^* = 2.6$. The corresponding quantities are
   $q_1(2.6, 2.6) = q_2(2.6, 2.6) = 0.48$. and the profits are
   $\pi_1(2.6, 2.6) = \pi_2(2.6, 2.6) = 0.768$.

   (b) Firm 1's new second-period profit maximization problem is:

   $$\max_{p_1} \pi_1(p_1, p_2) = p_1(1 - 0.3p_1 + 0.1p_2)$$

   Its first order condition and reaction function are:

   $$\frac{\partial \pi}{\partial p_1} = 1 - 0.3p_1 + 0.1p_2 - 0.3p_1 = 0 \Rightarrow p_1(p_2) = \frac{1 + 0.1p_2}{0.6}$$

For firm 2, the reaction function is unchanged from part (a), so solving simultaneously gives

$$p_1(p_2(p_1)) = \frac{1}{0.6} + \frac{0.1}{0.6}\frac{1.3 + 0.1p_1}{0.6} = 1.67 + 0.36 + 0.028p_1$$

Solving for p_1 and the other variables gives $p_1^* = 2.09, p_2^* = 2.52$, $q_1(2.09, 2.52) = 0.625$, $q_2(2.09, 2.52) = 0.453$. The (operating) profits are $\pi_1 = 2.09 \times 0.625 = 1.31$ and $\pi_2 = (2.52 - 1)0.453 = 0.69$.

(c) In stage 1, firm 1 anticipates that if it does not buy the machine, the stage 2 outcome will be as in (a), whereas if it does buy, the stage 2 outcome will be as in (b). If it does buy, the net profit for firm 1 is $\Pi_1 = 1.31 - 0.5 = 0.81$, so the machine is just barely worthwhile relative to the 0.768 outcome of part (a).