ECON 321, Professor Hogendorn

Problem Set 4

1. *Deterrence.* There is an industry with inverse demand curve $p(Q) = 100 - 4Q$. There is a first-mover firm that has marginal cost of 30 to build capacity and marginal cost of 10 to use capacity. There is an entrant firm that can pay entry cost e=90 to enter the industry and then has marginal cost of 30.

   First some preliminary results. If the two firms compete as Cournot competitors, their Cournot equilibrium operating profits are:

   $$\pi_1 = \frac{(130 - 2c_1)^2}{36} \qquad \pi_2 = \frac{(40 + c_1)^2}{36}$$

   where c_1 is the relevant marginal cost of firm 1.

   It also helps to know that if firm 1 had a monopoly in this market, its monopoly quantity and profit would be

   $$q_m = \frac{100 - c_1}{8} \qquad \pi_m = \frac{(100 - c_1)^2}{16}$$

   The game the firms play is that in stage 1, firm 1 chooses a capacity k_1 at marginal cost 30, and then in stage 2 firm 2 chooses whether to enter and the firms then compete as Cournot competitors. In stage 2, firm 1 can produce at marginal cost 10 up to its capacity k_1 and at marginal cost 40 beyond its capacity.

   (a) Write down the stage 2 problem of firm 2 formally, show the first order condition, and continue far enough to show that firm 2's reaction function is

   $$q_2(q_1) = \frac{70 - 4q_1}{8}$$

(b) Draw a Cournot reaction function diagram for the two firms, showing firm 1's two reaction functions for its two different costs. Find each firm's profit at the two potential Cournot equilibria.

(c) To proceed further, you need to find firm 2's profit when firm 1 has already committed to a particular quantity. Show that this is

$$\pi_{\mathrm{SF}} = \frac{(70 - 4\mathrm{q}_1)^2}{16}$$

This can be called the Stackelberg follower profit. Why are we finding it here since firm 1 cannot commit to the quantity produced?

(d) Can firm 1 blockade entry?

(e) What level of k_1 would deter entry?

(f) Is it more profitable for firm 1 to deter entry or to do nothing and allow Cournot competition to occur?

Review problem:

2. *Grandmammy.* Your grandmammy built a factory years ago with capacity $\bar{k}_1$. She knew that nowadays, the market inverse demand would be $p = 10 - q_1 - q_2$, where q_1 is the production of her (now your) factory and q_2 is the production that *may* come from an upstart factory that *may* be started by firm 2.

Your grandmammy and you have a cost of 2 to build a unit of capacity and a variable cost of 1 to produce output, so your total cost is $TC_1 = 2\bar{k}_1 + q_1$ if $q_1 \leq \bar{k}_1$ and $TC_1 = 3q_1$ if $q_1 > \bar{k}_1$.

The entrant firm 2 has a cost of 2 per unit of output q_2 and a fixed cost of 7.

(a) Find your reaction function if grandmammy built a huge factory.

(b) Find your reaction function if grandmammy built a size 0 factory.

(c) Find firms 2' reaction function.

(d) Find the Cournot equilibria for the huge factory and tiny factory cases. What are firms 2' profits in each case?

Answers to review problems:

2. *Grandmammy_a.*

   (a) With a huge factory, firm 1 solves

$$\max_{q_1} \pi_1 = (10 - q_1 - q_2)q_1 - 2\bar{k}_1 - 1q_1$$

   This gives first order condition

$$\frac{\partial \pi_1}{\partial q_1} = 10 - q_1 - q_2 - q_1 - 1 = 0$$

   and reaction function

$$q_1(q_2) = \frac{10 - q_2 - 1}{2} = 4.5 - \tfrac{1}{2}q_2$$

   (b) With $\bar{k}_1 = 0$, firm 1 solves

$$\max_{q_1} \pi_1 = (10 - q_1 - q_2)q_1 - 3q_1$$

   This gives first order condition

$$\frac{\partial \pi_1}{\partial q_1} = 10 - q_1 - q_2 - q_1 - 3 = 0$$

   and reaction function

$$q_1(q_2) = \frac{10 - q_2 - 3}{2} = 3.5 - \tfrac{1}{2}q_2$$

(c) Firm 2 always has MC=2. Since its problem is otherwise the same, we know that its reaction function will be

$$q_2(q_1) = \frac{10 - q_1 - 2}{2} = 4 - ½q_1$$

(d) For the huge factory case, solving the reaction functions simultaneously gives

$$q_1 = 4.5 - ½(4 - ½q_1) \Rightarrow \frac{3}{4}q_1 = 4.5 - 2 \Rightarrow q_1 = 3.33$$

Then $q_2 = 2.33, p = 10 - 3.33 - 2.33 = 4.33$, and Firm 2's profit is $\pi_2 = (4.33 - 2)\times 2.33 - 7 = -1.6.$

For the small factory case, solving the reaction functions simultaneously gives

$$q_1 = 3.5 - ½(4 - ½q_1) \Rightarrow \frac{3}{4}q_1 = 3.5 - 2 \Rightarrow q_1 = 2$$

Then $q_2 = 3, p = 10 - 2 - 3 = 5$, and Firm 2's profit is $\pi_2 = (5 - 2)\times 3 - 7 = 2.$

Thus, we know that there is some factory size that would just deter Firm 2 from entering the market.