ECON 321, Professor Hogendorn

Problem Set 4 Answers

1. *Deterrence.*

   (a) Firm 2 solves

   $$max_{q_2} \pi_2 = (100 - 4q_1 - 4q_2 - 30)q_2$$

   Its first order condition is

   $$\frac{\partial \pi_2}{\partial q_2} = 100 - 4q_1 - 4q_2 - 30 - 4q_2 = 0$$

   Collecting terms gives

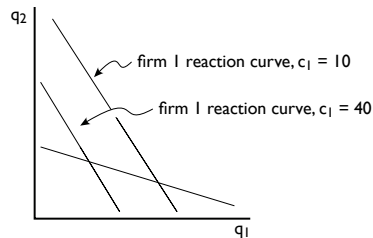 $$70 - 4q_1 - 8q_2 = 0$$

   from which you can obtain

   $$q_2(q_1) = \frac{70 - 4q_1}{8}$$

   (b) Using the formulas given at the start of the problem, and plugging in the marginal costs, gives:

   with $c_1 = 10$, $\pi_1 = 336.11$ and $\pi_2 = 69.44$.

   with $c_1 = 40$, $\pi_1 = 69.44$ and $\pi_2 = 177.78$

   The diagram is like this:

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(c) The profit of firm 2 is

$$\pi_{\text{SF}} = (100 - 4q_1 - 4q_2(q_1) - 30)q_2(q_1)$$

Substituting in the reaction function found in part (a) gives

$$\pi_{\text{SF}} = \left(70 - 4q_1 - 4\left(\frac{70 - 4q_1}{8}\right)\right)\frac{70 - 4q_1}{8}$$

This can be simplified to

$$\pi_{\text{SF}} = \left(\frac{70 - 4q_1}{2}\right)\frac{70 - 4q_1}{8}$$

Even though firm 1 can't *commit* to q_1, it *can* commit to having a low marginal cost up to whatever capacity it chooses. Then when it engages in Cournot competition, it will have an incentive to produce up to that capacity due to the low marginal cost. This means that, in effect, it can determine its future stage 2 equilibrium quantity in stage 1.

(d) If firm 1 simply ignores firm 2 and behaves like a monopolist, it chooses $q_m = 7.5$ given by plugging $c_1 = 40$ into the q_m formula.

If $q_1 = 7.5$, then the Stackelberg follower operating profit from part (c) is $\pi_{SF} = 100$. This is more than enough to cover the entry cost of 90, so blockading is not possible.

(e) We can solve for $\pi_{SF} - e = 0$:

$$\frac{(70 - 4q_1)^2}{16} - 90 = 0 \Rightarrow 70 - 4q_1 = \sqrt{1440} \Rightarrow q_1 = 8.025$$

(f) Under Cournot, we saw from part (a) that firm 1 earns 69.44. If it deters entry, it will produce quantity 8.025 but that will be the total quantity in the market, so its profits will be

$$\pi_{\text{Det}} = (100 - 4 \times 8.025 - 40)8.025 = 224$$

Thus, there is a large gain to deterring entry, or to put it another way, the first mover advantage is very valuable in this case.

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