ECON 321, Assignment 7:
BP, Chapter 4: 4.1 Stackelberg

1. Read 4.1.1. We'll make an example from the section titled "Price Competition."

2. Use Mathematica to set up a simple Bertrand model. Demand is

$$q_1(p_1, p_2) = a_1 - 2p_1 + p_2 \qquad q_2(p_1, p_2) = a_2 - 2p_2 + p_1$$

Costs are $c_1 = c_2 = c$ (symmetric, constant marginal cost).

Make the profit functions. Take the first order conditions for both firms using the timing of the Stackelberg game, i.e. firm 1 chooses p_1 and then firm 2 observes p_1 and chooses p_2.

3. For the case of $a_1 = a_2 = 7$ and $c = 1$, find the equilibrium prices, quantities, and profits. How big is the first mover advantage/disadvantage?

4. Read 4.1.3 on commitment. What is an example of a first-mover firm committing either to price or to some other variable?

5. Value added assignment for presenters: Let's run the game a bit differently. Suppose there are 3 stages:

Stage 1: firm 1 chooses a_1. Since this is an investment in quality or marketing, it's not free. The cost of this is $c(a_1) = 0.1a_1^2$.

Stage 2: firm 2 observes what firm 1 chose, and chooses its own a_2, also paying costs $c(a_2) = 0.1a_2^2$.

Stage 3: The firms compete in *simultaneous* differentiated Bertrand competition and receive Bertrand profits (just like in Assignment 4, except that the a_i terms in the demand functions are whatever was chosen in stages 1 and 2).

Solve this game backward to find a subgame perfect Nash equilibrum. Is there a first or second mover advantage in choosing a_i? Can you calibrate the results to a real world competitive market share situation? For example, if firm 1 has 75% market share and firm 2 has 25% in real life, you should be able to calibrate the model just by making firm 2's stage 2 costs higher, e.g. $c(a_2) = 0.3a_2^2$ might do the trick.