

## **ECON 321, Assignment 13: BP, Chapter 3: 3.4 and 3.5**

1. Read 3.4. Use Mathematica to set up the following very simple Bertrand model. Demand is

$$q_1(p_1, p_2) = 10 - 2p_1 + p_2 \quad q_2(p_1, p_2) = 10 - 2p_2 + p_1$$

Costs are  $c_1 = c_2 = 1$ . Make the profit functions. Take the first order conditions for both firms. (This is all review.)

2. Now, take the cross-partial derivatives. You can do this by taking the derivative of the left-hand-side of firm 1's first order conditions with respect to  $p_2$  and vice versa for firm 2. These evaluate to numbers. Is this game one of strategic complements or strategic substitutes?

3. Read 3.5. Now go back to the profit functions and take the derivatives manually using the product rule. That is, if you have  $\text{profit}_1 = (p_1 - 1) \text{demand}_1$ , then you can manually write the derivative as

$$\text{FOC1modified} = (p_1 - 1) D[\text{demand}_1, p_1] + \text{demand}_1$$

Except we're going to insert a  $\lambda$  in front of the derivative of demand, so actually write

$$\text{FOC1modified} = (p_1 - 1) \lambda D[\text{demand}_1, p_1] + \text{demand}_1$$

Do the same for firm 2.

4. Find the Bertrand equilibrium prices, quantities, and profits using these modified first order conditions.

5. Plot the profits of firm 1 for the cases of  $\lambda$  varying from  $-0.2$  to  $+1.5$ .