ECON 321, Assignment 2:

Making the substitions $a = 10$, $b = 2$, and $d = 1$, and also subbing in the budget constraint, the utility function is

$$u = 10q_1 + 10q_2 + \frac{1}{2}\left(-2q_1^2 - 2q_2q_1 - 2q_2^2\right) + y - p_1q_1 - p_2q_2 \tag{1}$$

To maximize this function, we take the two first order conditions.

$$\frac{\partial u}{\partial q_1} = -p_1 - 2q_1 - q_2 + 10 = 0 \qquad \frac{\partial u}{\partial q_2} = -p_2 - q_1 - 2q_2 + 10 = 0 \tag{2}$$

These FOCs basically give you the inverse demands directly, they're just

$$p_1(q_1, q_2) = 10 - 2q_1 - q_2 \qquad p_2(q_1, q_2) = 10 - q_1 - 2q_2 \tag{3}$$

Solve these two equations simultaneously for demand functions:

$$q_1(p_1, p_2) = \frac{1}{3}\left(10 - 2p_1 + p_2\right) \qquad q_2(p_1, p_2) = \frac{1}{3}\left(10 + p_1 - 2p_2\right) \tag{4}$$

Consumer Surplus: The formula says that consumer surplus, found directly from the utility function since utility is measured in dollars, is

$$CS(q_1, q_2) = \frac{1}{2}\left(2q_1^2 + 2q_2q_1 + 2q_2^2\right) \tag{5}$$

Now if $p_1 = p_2 = 3$ we can find the optimal quantities from equation 4 above. They are $q_1^* = 2.33$ and $q_2^* = 2.33$. So new CS is

$$CS(q_1^*, q_2^*) = CS(2.33, 2.33) = 16.33 \tag{6}$$

And if the prices change to $p_1 = 2$ and $p_2 = 3$, then we can find the new optimal quantities, which are $q_1^* = 3, q_2^* = 2$. So the new CS is

$$CS(q_1^*, q_2^*) = CS(3, 2) = 19 \tag{7}$$

The fall in price caused a gain in CS of \$2.66.

Extension: Single Good Demand Curves: So this is all sensible, but by using the utility function directly to calculate CS, we've kind of moved away from the usual paradigm of the triangular area under the demand curve. Does that still work when there are multiple goods?

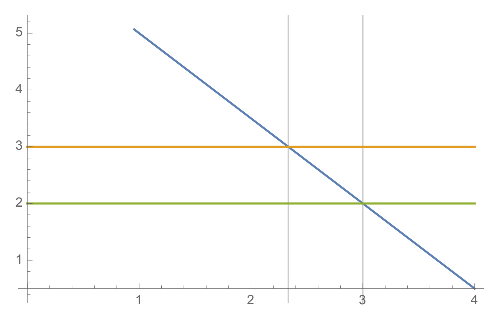
Let's keep the price $p_2 = 3$ just like above. If we substitute that in to the demand curves in equation 4, we can get a simple demand curve that shows how much q_1 to buy depending on the price p_1:

$$q_1(p_1) = 4.33 - 0.66p_1 \tag{8}$$

If you would rather work with inverse demand curves:

$$p_1(q_1) = 6.5 - 1.5q_1 \tag{9}$$

We can graph this function and show the two price/quantity points:



You can see how the fall in price of good 1 creates both a rectangle and a triangle of increased CS.

The area of the rectange is just $(3-2) \times 2.33 = 2.33$. And the area of the triangle is $\frac{1}{2}(3-2)(3-2.33) = 0.33$. So if you add up the gains, you get $2.33 + 0.33 = 2.66$

Hurray, that's the same as before, so this shows that a linear demand curve really can be thought of as the solution to a full utility max problem with the prices of the other goods held constant.