ECON 321, Assignment 3 Answers:
BP, Chapter 2: 2.2

The logic behind this assignment is that first we set up a single-product demand curve and then consider a similar demand system that has two products. But similar is a question here, because we are shifting the curves. A way to get the shift right is to anchor to two points, the $q = 0, p = 10$ point, and the point where $q_1 + q_2 = 3.33, p = 5$ that mimics the initial monopoly solution. That's where the coefficients come from for the demand curves.

The two-product firm solves

$$\max_{q_1,q_2} \pi(q_1, q_2) = \left(10 - 2q_1 - q_2 - 1\right) q_1 + \left(10 - q_1 - 2q_2 - 1\right) q_2 \tag{1}$$

If the firm chose $q_1 = q_2 = 1.67$ then this profit function would evaluate to $(5-1)1.67+(5-1)1.67 = 13.36$ which involves the same price and same profit as for the one-product case.

If the firm actually solves the maximization problem, it does so all at once, so it has 2 first order conditions to solve simultaneouly:

$$\frac{\partial \pi(q_1, q_2)}{\partial q_1} = 10 - 2q_1 - q_2 - 1 - 2q_1 - q_2 = 0 \tag{2}$$

$$\frac{\partial \pi(q_1, q_2)}{\partial q_2} = -q_1 + 10 - q_1 - 2q_2 - 1 - 2q_2 = 0 \tag{3}$$

These simplify to

$$9 - 4q_1 - 2q_2 = 0 \qquad 9 - 2q_1 - 4q_2 = 0 \tag{4}$$

Solving the first equation for q_1 gives $q_1 = \frac{9-2q_2}{4}$. Subbing that into the second equation gives $9 - \frac{9-2q_2}{2} - 4q_2 = 0$. This simplifies to $4.5 - 3q_2 = 0$ or $q_2^* = 1.5$. The problem is symmetric so we also have $q_1^* = 1.5$.

The price for both products is $p_1^* = p_2^* = 10 - 3 \times 1.5 = 5.5$ and the profit is $\pi^* = (5.5 - 1)1.5 + (5.5 - 1)1.5 = 13.5$. Notice that price is higher, total quantity is lower, and profit is higher than in the case of a single product.