ECON 321, Assignment 4:
BP, Chapter 3: 3.2

Let $p(q) = 10 - 2q$

Let $TC(q_i) = c_i q_i$ so $MC(q_i) = c_i$.

Then firm i's profit maximization problem is:

$$\max_{q_i} \pi_i = (10 - 2(q_i + q_{-i}))q_i - c_i q_i$$

Firm i's FOC (holding q_{-i} fixed) is:

$$\frac{\partial \pi_i}{\partial q_i} = 10 - 2q_i - 2q_{-i} - 2q_i - c_i = 0 \tag{1}$$

$$= 10 - c_i - 4q_i - 2q_{-i} \tag{2}$$

This can be solved for firm i's ***best response function***

$$q_i(q_{-i}) = \frac{10 - c_i - 2q_{-i}}{4} \tag{3}$$

If we simplify so that there are just 2 firms, 1 and 2, then the reaction functions (3) simplify to just

$$q_1(q_2) = \frac{10 - c_1 - 2q_2}{4} \qquad q_2(q_1) = \frac{10 - c_2 - 2q_1}{4} \tag{4}$$

Solving simultaneously gives

$$q_1^* = \frac{10 - 2c_1 + c_2}{6} \qquad q_2^* = \frac{10 - 2c_2 + c_1}{6} \tag{5}$$

If both firms have marginal cost equal to 1, then

$$q_1^* = q_2^* = 1.5 \tag{6}$$

With these quantities $p^* = 10 - 2(1.5 + 1.5) = 4$. Then the profits are

$$\pi_1^* = \pi_2^* = (4 - 1)1.5 = 4.5 \tag{7}$$

The sum of the profits is 9.

If instead $c_1 = 1$ but $c_2 = 2$ we expect firm 2 to be disadvantaged. The Cournot equilibrium quantities are

$$q_1^* = 1.67 \qquad q_2^* = 1.17 \tag{8}$$

so the price is $p^* = 10 - 2(1.67 + 1.17) = 4.32$ and the profits are

$$\pi_1^* = (4.32 - 1)1.67 = 5.54 \qquad \pi_2^* = (4.32 - 2)1.17 = 2.71 \tag{9}$$

The sum of the profits is 8.25.

So the rise in costs at firm 2 makes firm 2 worse off on all dimensions, it makes firm 1 better off, it makes the industry as a whole worse off, and since price is higher it makes consumers worse off.